UPPER BOUNDS IN LOW-ENERGY SUSY

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Abstract

In the constrained MSSM one is typically able to restrict the supersymmetric mass spectra below roughly 1-2 TeV *without* resorting to the ambiguous fine-tuning constraint.

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1 Constrained MSSM (CMSSM)

Low-energy SUSY has been considered an attractive extension of the Standard Model (SM) ever since it was introduced over a decade ago. In the early days many expected supersymmetric masses to lie rather low, "just around the corner", often well within the reach of LEP and the Tevatron. Not finding SUSY signals there was consequently rather disappointing and one could hear from sceptics sarcastic comments that a SUSY discovery will always remain to be expected for the *next* round of accelerators, in a time-invariant manner. Theoretical arguments based on no fine-tuning limiting SUSY masses roughly below $1\,\mathrm{TeV}$ were greeted with even less trust. After all, theorists are known to be both creative and, at the same time, rather unwilling to give up their most beloved toys, as the continuing activity in alternatives to SUSY clearly shows.

In this talk I am going to show that, in the Minimal Supersymmetric Standard Model (MSSM) with a few sensible and relatively general assumptions, one is often able to limit the SUSY particle masses below about 1-2 TeV *by physical constraints alone* [1, 2], without having to resort to an ill-defined fine-tuning constraint. Furthermore, the assumptions that we make are actually typically also made in most phenomenological studies of the MSSM and are well-motivated by GUTs. I will call this framework the constrained MSSM (CMSSM) [1].

First, it is worth remembering that SUSY alone has been applied to particle physics in order to provide a sensible framework for GUTs, which are otherwise plagued by the (in)famous problems of naturalness and scale hierarchy. Without GUTs, or related attempts (like strings) to not only unify all interactions but also to close the gap between to Fermi scale and the only fundamental scale in high energy physics, the Planck scale, there is indeed little motivation to consider low-energy SUSY. Furthermore, precision measurements at LEP have provided us with a remarkable argument for gauge coupling unification within (even minimal) SUSY, while showing more than clearly that within the SM alone such unification does not take place [3]. We will thus require that gauge couplings unify which will, for our purpose, fix the unification scale M_X. By doing so we actually are not forced to assume the existence of any specific GUT. We will only assume that $\sin^2\theta_{\mathrm{w}}(M_X) = \frac{3}{8}$ which also holds in many phenomenologically viable superstring-derived models.

Second, if the idea of unification is to be taken seriously, then one should expect not only the gauge couplings to emerge from a common source but also the same to be true for the various mass parameters of low-energy SUSY. In particular, one typically assumes that all the mass terms of the scalars in the model, like the squarks, sleptons and the Higgs bosons, originate from one "common" source m_0. Similarly, the masses of the gauginos (the gluino, winos and bino) should be equal to the "common" gaugino mass $m_{1/2}$ at M_X. These two assumptions are certainly not irrefutable but are at least sensible. In addition, they result from the simplest minimal supergravity framework and the simplest choice of the kinetic potential. Furthermore, there is

at least some partial motivation for assuming the common scalar mass m_0 coming from experiment. The near mass degeneracy in the $K^0 - \bar{K}^0$ system implies a near mass degeneracy between $\widetilde{s}_L$ and $\widetilde{d}_L$ [4]. Similarly, some slepton masses have to be strongly degenerate from stringent bounds on $\mu \to e\gamma$ [4]. Needless to say, most phenomenological studies of SUSY rely on at least one of these two assumptions, at least for the sake of reducing the otherwise huge number of unrelated SUSY mass parameters. We also assume that the trilinear soft SUSY-breaking terms are equal to A_0 at M_X, although this assumption has actually almost no bearing here.

Furthermore, it has been long known that in SUSY there exists a remarkable "built-in" mechanism of radiative electroweak symmetry breaking (EWSB). When the Higgs mass-square parameters are run from the high scale down, at some point the Higgs fields develop vevs. We thus require that the conditions for EWSB be satisfied.

Having made these sensible and well-motivated assumptions, we can next derive complete mass spectra of all the Higgs and supersymmetric particles by running their 1-loop RGEs between M_X and m_Z. The spectra are parametrized in terms of just a few basic parameters which we conveniently choose to be: the top mass M_t, $\tan\beta$, $m_{1/2}$, m_0, as well as A_0. The parameters $|\mu|$ and B are determined through the conditions for EWSB, but the sign of μ remains undetermined. We thus consider both $\operatorname{sgn}\mu = \pm 1$. We also employ the full 1-loop effective Higgs potential.

Besides requiring that EWSB occur, we demand that all physical mass-squares remain positive. We impose mass limits from current direct experimental searches and include the requirement that the solutions provide a BR$(b \to s\gamma)$ consistent with CLEO data. Furthermore, we calculate the relic density of the lightest SUSY particle (LSP), demanding only that the LSP be neutral, and, from limits on the age of the Universe of 10 billion years, we demand that $\Omega_{LSP}h_0^2 < 1$. Those solutions which finally remain after all these cuts comprise the allowed parameter space of the CMSSM.

2 Upper Limits

We have explored wide ranges of parameters, as described in detail in Refs. [1] and [2]. Clearly, in general one expects the emerging patterns of the SUSY mass spectra and properties (mixings, *etc.*) to vary strongly with the input parameters. While this is indeed true to some extent, nevertheless certain universal features emerge.

These features are illustrated in Fig. 1 for $M_t = 170\,\mathrm{GeV}$. The region of small $m_{1/2}$ is always excluded by either direct experimental searches for SUSY at LEP (typically the strongest bounds come from chargino or Higgs mass limits) or at the Tevatron (gluino). In some cases, in particular for $|A_0|/m_0$ significantly above zero, the lighter stop becomes too light, and even tachyonic, for $m_0 \gg m_{1/2} \lesssim 100\,\mathrm{GeV}$. Also, for some but rare combinations of parameters, for either $m_0 \gg m_{1/2}$ or $m_{1/2} \gg m_0$ the conditions for EWSB fail to be satisfied.
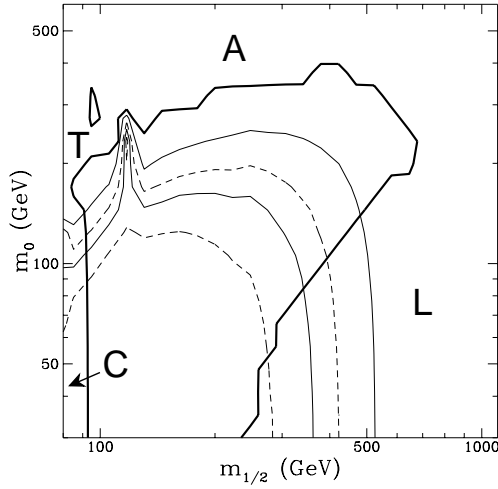
Figure 1: The regions of the $(m_{1/2}, m_0)$ plane consistent with low $\tan\beta$ b–τ mass uni-fication, given all the constraints of the CMSSM, for $M_t = 170\,\mathrm{GeV}$, $A_0/m_0 = 0$ and $\mu < 0$. Solutions outside the thick solid lines are excluded: on the left (small $m_{1/2}$) by the chargino mass bound (**C**) $m_{\chi^\pm} > 47\,\mathrm{GeV}$ and by tachyonic $\widetilde{t}$'s (**T**); on the right (large $m_{1/2} \gg m_0$) by charged LSP (**L**); and from above by the age of the Universe, *i.e.* $\Omega_\chi h_0^2 \leq 1$ (**A**). We also indicate the sub-regions selected by either the hypothesis of cold dark matter ($0.25 \lesssim \Omega_\chi h_0^2 \lesssim 0.5$, between thin solid lines) or the one of mixed dark matter ($0.16 \lesssim \Omega_\chi h_0^2 \lesssim 0.33$, between thin dashed lines).

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Furthermore, since we assume unbroken R-parity here, the LSP is stable and should be present as a relic in the Universe. There are strong arguments against charged exotic relics. We thus require the LSP to be neutral. This rules out a significant region of the $(m_{1/2}, m_0)$ parameter space corresponding to $m_{1/2} \gg m_0$ where the LSP is the lighter stau $\tilde{\tau}_1$. (Also $\tilde{e}_R$ and $\tilde{\mu}_R$ are not much heavier there.)

In the remaining region allowed by all the conditions listed above it is the lightest neutralino χ that is the LSP. (The sneutrino, another neutral sparticle, is the LSP in the region of small $m_{1/2}$ which is now completely excluded experimentally.) This is quite remarkable given the fact that the neutralino is a very attractive candidate for the dark matter (DM) in the Universe for which there seems to be inescapable need among astrophysicists [5]. Equally remarkable and non-trivial is the fact that χ comes out mostly bino-like which is essentially a necessary condition if one expects the neutralino to be a significant component of DM in the Universe. (The neutralino with a significant higgsino admixture has invariably very small relic abundance [5].) The only exceptions to this general rule can be found in some relatively rare case in very tiny regions of the $(m_{1/2}, m_0)$ on the border of the region where the conditions for the EWSB cannot be satisfied.

The fact that the lightest neutralino of bino-type comes out in the CMSSM as the unique neutral candidate for the LSP is not only interesting in itself. It also leads to a very remarkable upper bound on both $m_{1/2}$ and m_0. This comes about as follows. The neutralino relic density ρ_χ depends on how many neutralinos have pair-annihilated in the early Universe. Their number effectively froze when the expansion rate exceeded the annihilation rate. In order to calculate the neutralino relic density one thus needs to include all the annihilation channels of the neutralinos into ordinary particles, which we do. Since all the masses and mixings are determined in the CMSSM in terms of the basic independent parameters listed above, one can also express in terms of them the neutralino relic abundance $\Omega_\chi h_0^2$ (which is the neutralino relic density in units of the critical density times the squared reduced Hubble constant). The key point is that any significant contribution to the total mass-energy density of the Universe would have affected its evolution. In particular, the greater the total density the faster the Universe expands and the more quickly it reaches its present size. The age of the Universe, which is known to be *at least* 10 billion years, then puts an upper limit $\Omega_\chi h_0^2 < 1$. This is shown in Fig. 1. We see that *the whole* plane $(m_{1/2}, m_0)$ becomes limited within a few hundred GeV.

The dominant effect is played here by the annihilation of the neutralinos into light fermion-antifermion pairs via the t-channel exchange of the lightest sfermion(s); roughly $\Omega_\chi h_0^2 \propto m_{\tilde{f}}^4 / m_\chi^2$ [5, 6], although including other final states affects the exact location of the bound.

It is interesting to explore how these bounds vary with different choices of the input parameters. We find that at least for small $\tan\beta \lesssim 2$ one is able to close almost the whole plane $(m_{1/2}, m_0)$, and therefore the whole SUSY spectrum, from *above* for any combinations of other parameters, except for the region of large Z-

pole enhancement ($m_0 \gg m_{1/2} \simeq 100\,\text{GeV}$). For larger values of $\tan\beta$ sometimes the conditions of EWSB cannot be satisfied in the regions of extreme $m_{1/2}$ or m_0 and very close to such regions the LSP is of higgsino-type. In such, relatively rare, cases one cannot close the plane ($m_{1/2}, m_0$) from above completely. It is remarkable that small $\tan\beta$ is strongly favoured by the simple unification of the b- and τ-Yukawa coupling unification, like in $SU(5)$ [7]. It was recognized several years ago that, unlike in the SM, in the MSSM the b- and τ-Yukawa running couplings meet at roughly the same mass scale at which the unification of the gauge couplings takes place [8]. In Ref. [2] we have studied in detail the various consequences of adding this sensible, but rather specific to $SU(5)$-type GUTs, assumption.

Can these upper bounds be improved? It is worth stressing that the assumption that the age of the Universe is at least 10 billion years is actually a rather conservative one. Many expect it to be no less than some 15 billion years which translates to $\Omega_\chi h_0^2 \lesssim 0.25$ and much tighter bounds on the parameters $m_{1/2}$ and m_0. Another attractive hypothesis is the one of cosmic inflation which predicts $\Omega = 1$ in which case most of the matter in the Universe must most likely hide in the form of DM. Two scenarios have attracted a lot of attention. In the purely cold DM (CDM) scenario the neutralino would constitute most of DM in the (flat) Universe in which case the range $0.25 \lesssim \Omega_\chi h_0^2 \lesssim 0.5$ would be favored. More recently (after COBE), a mixed CDM+HDM picture (MDM) became more popular as it apparently fits the astrophysical data better than the pure CDM model. In the mixed scenario one assumes about 30% HDM (like light neutrinos with $m_\nu \simeq 6\,\text{eV}$) and about 65% CDM (bino-like neutralino), with baryons contributing the remaining 5% of the DM. In this case the favored range for $\Omega_\chi h_0^2$ is approximately given by $0.16 \lesssim \Omega_\chi h_0^2 \lesssim 0.33$. Both ranges are plotted in Fig. 1. It is clear that their effect is to significantly reduce the allowed parameter space from *both above and below*. Consequently, the allowed mass ranges of the various SUSY (and Higgs) particles become much more restricted and, unfortunately, typically beyond the reach of LEP II and the upgraded Tevatron. More details can be found in Refs. [1] and [2].

3   Conclusions

I have shown that, in the framework of Constrained MSSM (which is the MSSM with a few well-motivated assumptions stemming from grand unifications), one can often limit the SUSY particle masses below about 1-2 TeV *by physical constraints alone*. I have not used the ill-defined fine-tuning constraint *at all*. It certainly still makes sense to take it into account in expressing our expectations as to where SUSY might be realized. But relatively general physical constraints now do not allow us to push SUSY into a multi- TeV region even if we wanted. Especially with the improving knowledge of the top mass and the age of the Universe we soon will be able to make a definite statement, based purely on physics criteria, that (minimal) low-energy SUSY is either realized roughly below 1 TeV or is not realized in Nature at all.

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